# Mixed CS levels and strip geometry for abelian 3d N=2

Shi Cheng

2010.15074 and work in progress

April 20

- 3d $\mathcal{N}=2$ gauge theory
- mirror symmetry & CS levels.
- brane webs & CS levels

# 3d $\mathcal{N}=2$ Chern-Simons matter theory, $U(1)^n$

- Vector multiplet, $\ni$ $A_\mu$ gauge field.

- chiral multiplet $\leftarrow$ matter

CS term,

$$\mathcal{L}_{CS} = k_{ab} \int d^4\theta \; \Sigma_a \cdot V_b .$$

$k_{ab}$: mixed CS levels

$V_b$: vector

$$\Sigma_a = \epsilon^{\alpha\beta} \bar{D}_\alpha \bar{D} V_a$$

FI term,

$$\mathcal{L}_{FI} = \zeta_a \int d^4\theta \; V_a$$

$\zeta_a$: FI parameters

## Quiver diagrams :

$n_c$ gauge group $SU(n_c)$

$\boxed{n_f}$ flavor symmetry $SU(n_f)$

$\longrightarrow$ chiral multiplet

$1 \longrightarrow \boxed{1}$ $U(1) + 1\ F$ ← chiral multiplet in $\square$ rep. of gauge group $U(1)$

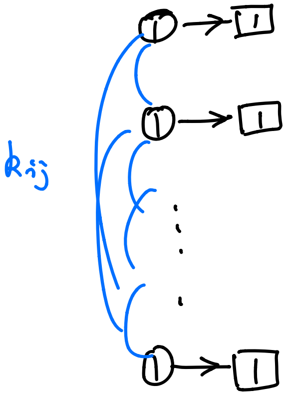
$1 \longleftarrow \boxed{1}$ $U(1) + 1\ AF$ ← chiral multiplet in $\bar{\square}$ rep of gauge group $U(1)$

$\boxed{1} \longrightarrow 1 \longrightarrow \boxed{1}$ $U(1) + 1\ F + 1\ AF$

$1 \rightrightarrows \boxed{2}$ $U(1) + 2\ F$

## Mixed CS levels $k_{ij}$ :

The coupling between gauge fields, $k_{ij} \int_{\mathbb{R}^3} A_i \wedge F_j$




building blocks $(1) \to \boxed{1}$ are coupled together by mixed CS couplings.

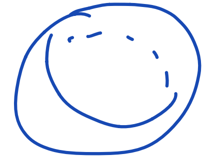
$\mathcal{T}_{A,N}$ theory :

- gauge group $U(1)^N$,
- $N$ fundamental chirals $NF$
- mixed CS level, $k_{ij}$
- FI parameter, $\xi_i$
- real mass parameter $m_i$

$A$: abelian

$N$: # of chirals.

## Sphere partition function :

put 3d $\mathcal{N}=2$ on  $S_b^3$

Localization $\rightsquigarrow$

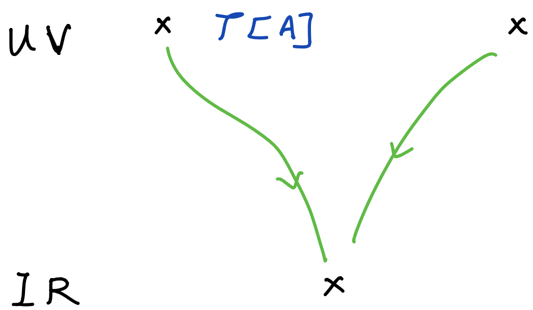
$$Z^{3d\,\mathcal{N}=2}_{S_b^3} = \int \underbrace{\prod_{i=1}^{N} dx^i}_{\text{integral over bosonic field in } V_a\text{.}} \prod_{i,j=1}^{N} \underbrace{S_b\Big(x_i \pm \frac{iQ}{2} + \overbrace{\frac{u_i}{2}}^{\text{real mass}}\Big)}_{\text{chirals}} \underbrace{e^{-i k_{ij} x_i x_j}}_{\text{CS term}} \cdot \underbrace{e^{2\pi i \xi x_i}}_{\text{FI}}$$

## Superpotential $\widetilde{W}^{\text{eff}}$ :

$$Z_{S_b^3} \sim \int dx \; e^{\frac{1}{g_s} \widetilde{W}^{\text{eff.}}(k_{ij},\, \xi_i,\, x)} \quad , \quad \text{when } g_s \to 0 .$$

- Superpotential $\widetilde{W}^{\text{eff}}$ determines the 3d theory, which is similar to prepotential $\mathcal{F}_{4d\,\mathcal{N}=2}$ in Seiberg – Witten theory

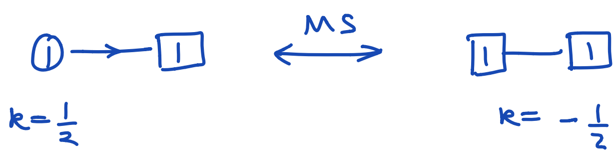
# Mirror Symmetry




$g \to 0$

$g \to \infty$, 3d QCD.

MS vs Fourier transformation on partition function,




MS transformation :

$$\int dy \; e^{-\frac{i\pi}{2} y^2} \, e^{2\pi i \left(\frac{iQ}{4} - i\right) y} \, S_b\!\left(\frac{iQ}{2} - y\right) \overset{MS}{=\!=} e^{\frac{i\pi}{2}\left(\frac{iQ}{2} - z\right)^2} S_b\!\left(\frac{iQ}{2} - z\right)$$

Contribution of each chirals can be replaced by an integral

$$S_b\left(\frac{iQ}{2} - z\right) \xrightarrow[\text{transf.}]{\text{mirror}} \int dy \; \boxed{//} \; S_b\left(\frac{iQ}{2} - y\right)$$

$$-\boxed{1} \;\rightarrow\; -(1)-\boxed{1}$$

Examples :

1- time :

$$\underset{k}{(1)} - \boxed{1} \xrightarrow{MS} (1) - \left((1)' - \boxed{1}\right) = \underset{k'}{(1)'} - \boxed{1}$$

2- time :

$$\underset{k}{(1)} - \boxed{1} \xrightarrow{MS} (1) - \left((1)' - \boxed{1}\right) \xrightarrow{MS} (1) - \left((1)' - \left((1)'' - \boxed{1}\right)\right)$$

$$\parallel$$

$$\underset{k''}{(1)''} - \boxed{1}$$

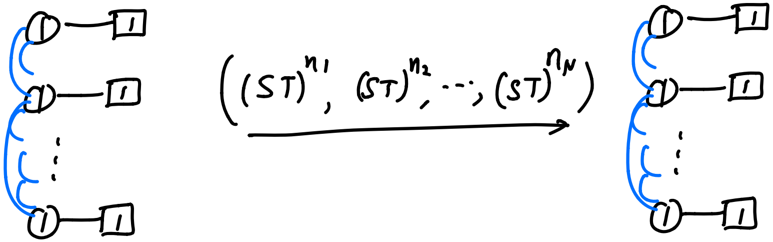
MS gives to new theories w/ different CS levels, and FI parameters

3- time :

$$\underset{k}{(1)} - \boxed{1} \xrightarrow{MS} \cdot \xrightarrow{MS} \cdot \xrightarrow{MS} \underset{k}{(1)} - \boxed{1} \,,$$

we go back to the original theory, it is because, mirror symmetry as ST- transf. on Lagrangian, $(ST)^3 = 1$ [Witten]

We can perform MS on $T_{A,N}$ theory, acting on each node $\textcircled{1}_i$:




$k_{ij}\ ,\ s_i$ $\qquad\qquad$ $k_{ij}'\ ,\ s_i'$

$$T_{A,N} \xrightarrow{ST} T_{A,N}'$$

We use $(n_1, n_2, \cdots, n_N)$ to denote $((ST)^{n_1}, (ST)^{n_2}, \cdots (ST)^{n_N})$

$n_i$ : # of $ST$-transformations on $i$-th gauge group $U(1)_i$,

For $T_{A,N}$ theory, we get a mirror transformation group.

$$\cancel{M}(T_{A,N}) = \{ (n_1, n_2, \cdots, n_N) \mid n_i = 0, 1, 2 \}$$

$$(i_1, i_2, \cdots, i_N) + (n_1, n_2 \cdots, n_N) = (n_1 + i_1, n_2 + i_2, \cdots, n_N + i_N)$$

$$(n_1, n_2, \cdots) \longrightarrow J[(n_1, n_2, \cdots)]$$

Each element $(n_1, n_2, \cdots)$ gives, a mirror dual theory

<u>Example</u>: $J_{A,2}$,

$$\mathcal{N}(J_{A,2}) = \{(0,0), (0,1), (1,0), (0,2), (2,0), (1,2), (2,1), (1,1), (2,2)\}$$

We have <u>commutative diagram</u>

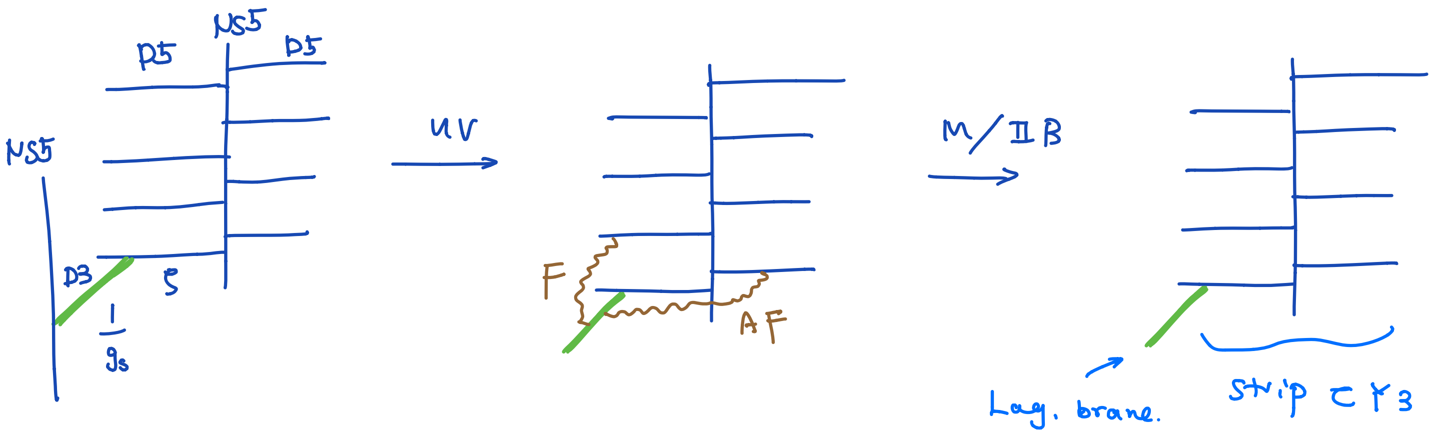
$$\begin{tikzcd}
J[(2,0)] \arrow[r, "(0,1)"] & J[(2,1)] \arrow[r, "(0,1)"] & J[(2,2)] \\
J[(1,0)] \arrow[u, "(1,0)"] \arrow[r, "(0,1)"] & J[(1,1)] \arrow[u, "(1,0)"] \arrow[r, "(0,1)"] & J[(1,2)] \arrow[u, "(1,0)"] \\
J[(0,0)] \arrow[u, "(1,0)"] \arrow[r, "(0,1)"] & J[(0,1)] \arrow[u, "(1,0)"] \arrow[r, "(0,1)"] & J[(0,2)] \arrow[u, "(1,0)"]
\end{tikzcd}$$

- Mirror symmetry preserves sphere partition function. ✓
- Mirror symmetry presers vortex partition function ✓

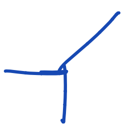
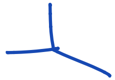
$$Z_{S^3_b} \sim \sum_i Z^{\text{vortex}}_{\mathbb{R}^2\times S^1}(q) \cdot Z^{\text{vortex}}_{\mathbb{R}^2\times S^1}(q^{-1})$$

# Application

$\boxed{n_{af}} \to (1) \to \boxed{n_f}$ , $U(1) + n_f\, F + n_{af}\, AF$ can be realized by brane webs in IIB string theory.



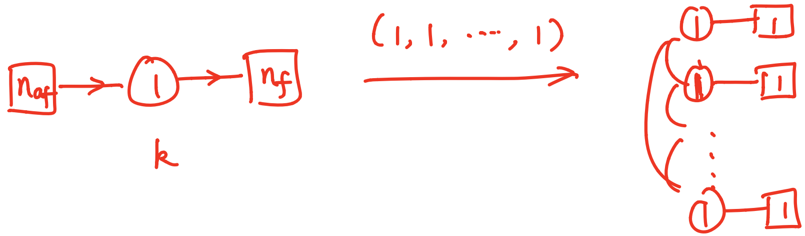

Since branes have tensions,

 is actually  or 

We can draw brane webs more precisely.

toric diagram for resolved conifold,

- $U(1) + n_f F + n_{af} AF$, can be transformed to $T_{A,N}$ theory by mirror symmetry,




$T_{A,\, n_f + n_{af}}(k_{ij}, \xi)$

$$Z^{\textcircled{1}-\boxed{N}}_{S_b^3} = \int dx\, e^{-i\pi k x^2 + 2\pi i \xi x} \underbrace{\prod_{i=1}^{N} s_b\left(\frac{iQ}{2} + x + \frac{m_i}{2}\right)}$$

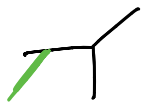
$$\xrightarrow{(1,1,\cdots,1)} \int \prod_{i=1}^{N} dy_i\, e^{-\pi i k_{ij} y_i y_j + 2\pi i \xi_i y_i} \prod_{i=1}^{N} s_b\left(\frac{iQ}{2} - y_i\right) = Z^{T_{A,N}}_{S_b^3}$$

We can continue the mirror transformation $\mathcal{M}(\mathcal{T}_{A,N})$ on this $\mathcal{T}_{A,N}$ given by $U(1) + n_f\, F + n_{af}\, AF$, and get a group of $\underbrace{\text{integer effective mixed CS level}}$. $\{ k^{eff}_{ij} \}$

parity anomaly — renormalization from one-loop contribution.

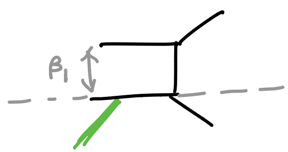
## Examples:

$\underset{k}{(1)} - \boxed{1}$ :



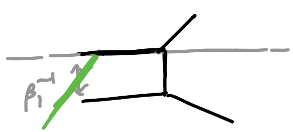
$\mathbb{C}^3$, $k^{eff} = k + \frac{1}{2}$,

$\underset{k}{(1)} - \boxed{2}$ :




$$k^{eff}_{ij}(2,0) = \begin{bmatrix} k+\frac{1}{2} & 1 \\ 1 & 0 \end{bmatrix}$$

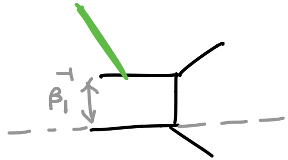
$\downarrow$ flip $\qquad\qquad \downarrow (0,1)$



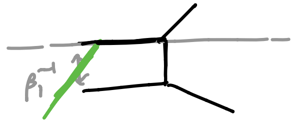

$$k^{eff}_{ij}(2,1) = \begin{bmatrix} k & -1 \\ -1 & 0 \end{bmatrix}$$

- $\beta \sim e^{im}$, hence, flip is $m \to -m$.

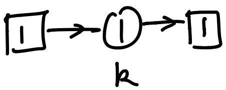
- we can also change the position of Lagrangian brane (in green).

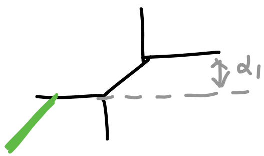



$$k_{ij}^{eff}(0,2) = \begin{bmatrix} 1 & 1 \\ 1 & 1+k \end{bmatrix}$$



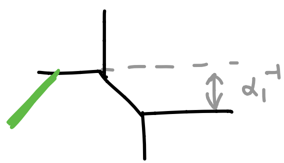

$$k_{ij}^{eff}(1,2) = \begin{bmatrix} 0 & -1 \\ -1 & k \end{bmatrix}$$




$$k_{ij}^{eff}(2,1) = \begin{bmatrix} k & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Big\downarrow (0,2)$$

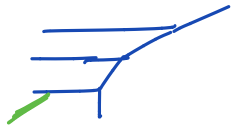



$$k_{ij}^{eff}(2,0) = \begin{bmatrix} k+1 & -1 \\ -1 & 1 \end{bmatrix}$$

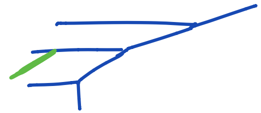
⋮ ⋮

$\textcircled{1} \longrightarrow \boxed{3}$

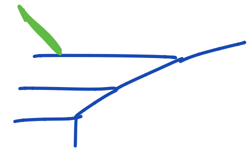
k



T[(2,0,0)]

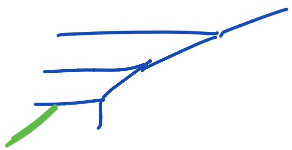


T[(0,2,0)]



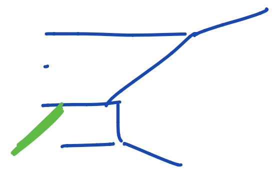
T[(0,0,2)]

For the first one,



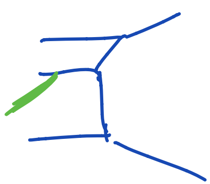
T(2,0,0)

$\xrightarrow{(0,1,0)}$



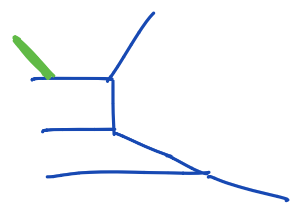
T(0,2,0)

$\downarrow$ $\downarrow$



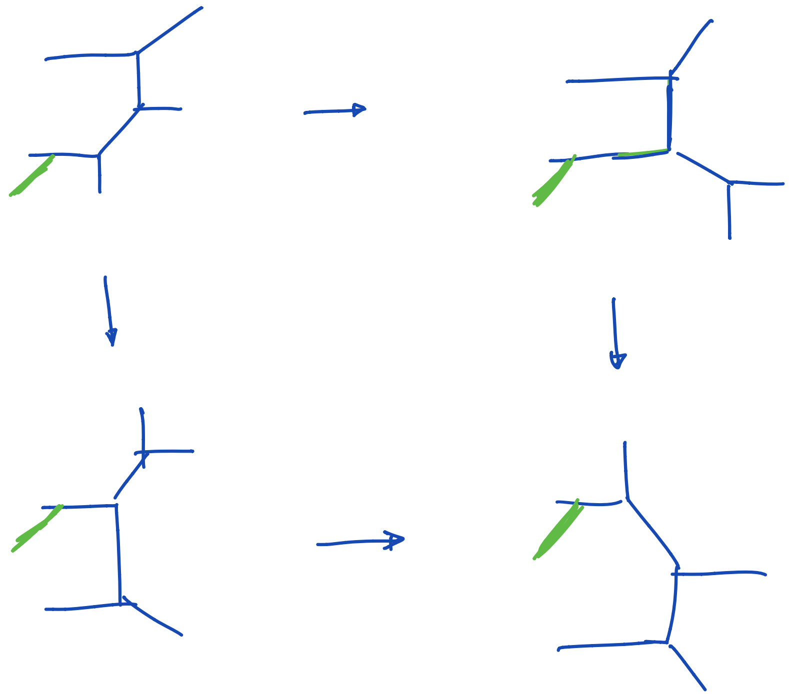
T (2,0,1)

$\longrightarrow$



T (2,1,1)

$\boxed{1} \rightarrow \underset{k}{1} \rightarrow \boxed{2}$



- Integer effective CS level $\{k^{\text{eff}}_{ij}\}$ are one to one corresponding to brane webs w/ a Lagrangian brane.

Thank you.